P 148 Exercises

[1.1] $2\vec{a} + \vec{x} = 3\vec{b}$

$$\vec{x} = 3\vec{b} - 2\vec{a}$$

$$= 3\langle 7, 9, -8\rangle - 2\langle 4, -2, 5\rangle$$

$$= \langle 13, 31, -34\rangle$$

$\therefore\ x = \langle 13, 31, -34\rangle$

[1.2] $4\vec{x} - \vec{a} = 3\vec{a} - 4\vec{b} + 2\vec{x}$

$$2\vec{x} = 4\vec{a} - 4\vec{b}$$

$$x = 2(\vec{a} - \vec{b})$$

$$= 2\begin{bmatrix} 4-7 \\ -2-9 \\ 5+8 \end{bmatrix}$$

$$= 2\begin{bmatrix} -3 \\ -11 \\ 13 \end{bmatrix}$$

$\therefore\ x = \langle -6, -22, 26\rangle$

P 148 ctd

[2] $\vec{a} = \langle 1,1,0 \rangle$, $\vec{b} = \langle 1,0,1 \rangle$, $c = \langle 0,1,1 \rangle$, $\vec{p} = \langle 5,6,7 \rangle$

$$\vec{p} = l\vec{a} + m\vec{b} + n\vec{c}$$

$$l\begin{bmatrix}1\\1\\0\end{bmatrix} + m\begin{bmatrix}1\\0\\1\end{bmatrix} + n\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}5\\6\\7\end{bmatrix}$$

$$\begin{bmatrix} l + m \quad = 5 \\ l \quad + n = 6 \\ m + n = 7 \end{bmatrix} \equiv \begin{bmatrix} 1 & 1 & 0 & 5 \\ 1 & 0 & 1 & 6 \\ 0 & 1 & 1 & 7 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\therefore \quad 2\vec{a} + 3\vec{b} + 4\vec{c} = \vec{p}$$

P 149

[4] $\vec{a} = \langle 2,-1,-5 \rangle$, $\vec{b} = \langle 3x, 6, 4y-2 \rangle$, $c = \langle z-1, 2, z+1 \rangle$

[4.1] $\vec{a} \parallel \vec{b}$ iff $\vec{a} = t\vec{b}$, $t \in \mathbb{R}$.

$$t\begin{bmatrix}3x\\6\\4y-2\end{bmatrix} = \begin{bmatrix}2\\-1\\-5\end{bmatrix} \Rightarrow t = -\frac{1}{6}$$

then

$$-\frac{1}{6}\begin{bmatrix}3x\\6\\4y-2\end{bmatrix} = \begin{bmatrix}2\\-1\\-5\end{bmatrix} \Rightarrow \begin{bmatrix}3x\\6\\4y-2\end{bmatrix} = \begin{bmatrix}-12\\6\\30\end{bmatrix} \Rightarrow \begin{matrix} x = -4 \\ y = 8 \end{matrix}$$

$$\therefore \quad x = -4, \ y = 8$$

[4,2] $\vec{a} \perp \vec{c}$ iff $\vec{a} \cdot \vec{c} = 0$

$$\begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} \cdot \begin{bmatrix} z-1 \\ 2 \\ z+1 \end{bmatrix} = 2(z-1) + (-1)(2) + (-5)(z+1)$$

$$= 2z - 2 - 2 - 5z - 5$$

$\vec{a} \perp \vec{c}$ iff $-3z - 9 = 0$

$$z = -3$$

$\therefore z = -3$

[5] $\vec{a} = \langle x, 4, 6 \rangle$, $\vec{b} = \langle 2, y, 6 \rangle$, $\vec{c} = \langle 2, 4, z \rangle$. All $\perp$.

$\vec{a} \cdot \vec{b} = 0 \quad \Rightarrow \quad 2x + 4y + 36 = 0 \quad \Leftrightarrow \quad x + 2y + 18 = 0$

$\vec{a} \cdot \vec{c} = 0 \quad \Rightarrow \quad 2x + 16 + 6z = 0 \quad \Leftrightarrow \quad x + 3z + 8 = 0$

$\vec{b} \cdot \vec{c} = 0 \quad \Rightarrow \quad 4 + 4y + 6z = 0 \quad \Leftrightarrow \quad 2y + 3z + 2 = 0$

Then
$$\begin{bmatrix} 1 & 2 & 0 & -18 \\ 1 & 0 & 3 & -8 \\ 0 & 2 & 3 & -2 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 0 & -12 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4/3 \end{bmatrix}$$

$\therefore \vec{a} = \langle -12, 4, 6 \rangle$, $\vec{b} = \langle 2, -3, 6 \rangle$, $\vec{c} = \langle 2, 4, \frac{4}{3} \rangle$

[6] $\vec{a} = \langle 1, 2, -3 \rangle$, $\vec{b} = \langle 2, -1, -2 \rangle$, $\vec{x} = \langle x_1, x_2, x_3 \rangle$

where $\vec{x} \perp \vec{a}$ and $\vec{x} \perp \vec{b}$. Get x_1, x_2, x_3.

$$\vec{x} \perp \vec{a} \equiv \vec{x} \cdot \vec{a} = 0 \equiv x_1 + 2x_2 - 3x_3 = 0$$

$$\vec{x} \perp \vec{b} \equiv \vec{x} \cdot \vec{b} = 0 \equiv 2x_1 - x_2 - 2x_3 = 0$$

$$\begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & -7/5 & 0 \\ 0 & 1 & -4/5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $x_3 = t$, $t \in \mathbb{R}$

$x_2 = \frac{4}{5} t$

$x_1 = \frac{7}{5} t$

P 149 ctd

[7] $\vec{a} = \langle 2, -2, 1 \rangle$ and $\vec{b} = \langle 2, 3, -4 \rangle$

[7.1] $\vec{c} = \vec{b} - k\vec{a}$, $k \in \mathbb{R}$. ∌ $\vec{a} \perp \vec{c}$. get k and $\vec{c}$.

$$\vec{a} \perp \vec{c} \iff \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix} - k \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right) = 0$$

$$\iff \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 - 2k \\ 3 + 2k \\ -4 - k \end{bmatrix} = 0$$

$$\iff 2(2-2k) - 2(3+2k) + (-4-k) = 0$$

$$\iff 4 - 4k - 6 - 4k - 4 - k = 0$$

$$\iff -9k = 6$$

$$\iff k = -\frac{2}{3}$$

$\therefore k = -\frac{2}{3}$

[7.2] Get $\vec{u} \ni |\vec{u}| = 3$ and $\vec{u} \perp \vec{a}$ and $\vec{u} \perp \vec{b}$. Let $\vec{u} = \langle u_1, u_2, u_3 \rangle$

$\vec{a} \perp \vec{u} \Rightarrow \vec{a} \cdot \vec{u} = 2u_1 - 2u_2 + u_3 = 0$

$\vec{b} \perp \vec{u} \Rightarrow \vec{b} \cdot \vec{u} = 2u_1 + 3u_2 - 4u_3 = 0$

$$\begin{bmatrix} 2 & -2 & 1 & 0 \\ 2 & 3 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{aligned} u_3 &= t \\ u_2 &= u_3 = t \\ u_1 &= \tfrac{1}{2} t \end{aligned}$$

since $|\vec{u}| = 3$, we have

$$u_1^2 + u_2^2 + u_3^2 = 9$$

$$\frac{t^2}{4} + t^2 + t^2 = 9$$

$$t^2 + 4t^2 + 4t^2 = 36$$

$$9t^2 = 36$$

$$t = 2$$

$$\boxed{\therefore \vec{u} = \langle 1, 2, 2 \rangle}$$